

Probability Default in Black Scholes Formula: A Qualitative Study

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Abstract: A default risk is the risk that a person or an organization will fail to make a payment that they have promised. There are many models that help us to analyze credit risk, such as Default Probability, Loss Given Default, and Migration Risk. All these models are important for evaluating credit risk, but the most important factor is the Probability of Default that is mentioned in this paper. This paper uses the Black Scholes formula for European call option to find the probability default of a firm. How d_2 in Black schools model became the probability default of a Merton model. Merton model is the structural model because it is using firm's value to inform the probability of firms default and here we are going to show the relationship between Black Scholes European call option and the probability of default of a firm. The main aim of this paper is to describe the factor that affects the default probability default using Black Scholes model for European Call option by the help of some examples.

Keywords: Black Scholes Model, Merton Model, Probability Default, Probability Distance

1. Introduction

Credit risk refers to the risk that borrower will default on any type of the debt by failing to make payment which it is obligated to do. For example a consumer may fail to make a payment due to mortgage loan, credit card, line of credit, or other loan. The credit risk became the popular in recent years, and various methods are using to measure the credit risk exposure. The firm's profitability changes with the business cycle. In general, in an expansion, demand is high and business is strong: firms have higher probability to profit and therefore fewer defaults will happen

The structural and reduced methods are popular method to measure the credit default risk but in this paper our focus is on only structural model. The structural model/approach aims to provide an explicit relationship between the capital structural and default risk. Merton Model was the first structural model and has served as the cornerstone for all the other structural models [6]. For understanding the Merton model we will go through Merton model in detail, and briefly introduce some importance. The Merton model is the structural model because it is using firm's value to inform the

probability of firms default. Merton (1974), a firm will expect default only when the value of the assets goes below a threshold value which is determined by its liabilities. If the value of a firm goes below a certain threshold, then the owners will put the firm to the debt holders.

Over the last few years, numbers of researchers have contribution of the Merton model. The first authors to examine the model carefully were practitioners employed by either KMV or Moody's. Crosbie and Bohn (2003) summarize KMV's default probability model [6]. Stein (2002) argues that KMV-Merton models can easily be improved upon [13]. Other papers, including Bohn, Arora and Korablev (2005), argued that KMV-Merton models capture all of the information in traditional agency ratings and well known accounting variables [3]. Duffie and Wang (2004) show that KMV-Merton probabilities have significant predictive power in a model of default probabilities over time, which can generate a term structure of default probabilities [7]. Farmen, Westgaard et al (2003) investigate the default probabilities and their comparative statics (default Greeks) in the Merton framework using the objective or 'real' probability measure [9]. Bohn (2000) surveys some of the main theoretical models of risky debt valuation that

built on Merton (1974) and Black and Cox (1976) [2]. In the Indian context, credit risk modelling has been attempted based on corporate bond ratings. Varma and Raghunathan (2000) analyze credit rating migrations in Indian corporate bond market to bring about greater understanding of its credit risk [14].

The Black Scholes Model (BSM) is the method of modelling derivatives prices was first introduced in 1973. In 1997, the importance of the model was recognised when Robert Merton and Myron Scholes were awarded the Nobel Prize for economics. Sadly, Fischer Black died in 1995. Otherwise he too would undoubtedly have one of the recipients of this prize [10]. Essentially, BSM formula shows us how to find the price of an option contract (call and put option) can be determined by using simple formula but here we are using only European call option. The formula for European call option is

$$C(S, T) = SN(d_1) - Ke^{-rT}N(d_2) \quad (1)$$

Where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (2)$$

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \quad (3)$$

S is the present price of the stock, K is the strike price, r is the free risk rate interest, σ is the variance parameter of the stock and N is the CDF function for a standard normal distribution.

Option is a contract that gives the owner the rights, but not obligation to buy (in case of call option) or sell (in case of put option) an asset at a fixed price K at a set date T . Hence, a call/put option owner has the rights to buy/sell the underlying asset at the specified contract price and date. In this paper we are assuming that the options are premium free. This is the price modeled using BSM formula, which is straightforward in the implementation for European style options. American options can be exercised before the maturity date, whereas European option can be exercise only the maturity date. In this paper we are discussing only European style Call Option of BSM approach. Here in this paper we are going to assume that:

Firm's value (V) corresponds to stock price (S), Firm's value debt (F) corresponds to exercise/strike price (K) and r is the expected growth on the firm's asset not risk free rate.

We will illustrate the importance of $N(d_1)$ in BSM approach. However, one of the useful and interesting is $N(-d_1)$ is the probability that this option (call option) will be stuck in the risk neutral not a real world. The $N(-d_1)$ is the idea behind the Merton Model to Credit risk.

Finally we will explain the various factors that affect the probability default by using BSM.

We will change one factor and others will be constant likewise we will explain with the help of figures to explain the default probability factors.

2. Credit Risk, Default Probability and Distance to Default

Credit risk measurement and management has become one of the most important topics in financial economics. The term default essentially means a debtor has not paid a debt. "Insolvency" is a legal term meaning that a debtor is unable to pay his/her debts.

A credit is the risk of loss due to a debtors non-payment of a loan or other line of credit (either the principal or interest (coupon) or both). It is the risk due to uncertainty in a counterparty's (also called an obligors or credits) ability to meet its obligations.

For example: A company is unable to repay amounts secured by a fixed or floating charges over the asset of the company, a business does not pay an employee's earnings when due, an insolvent firm won't return funds to a depositor. Credit risk modeling helps to estimate how much credit is at risk due to a default or changes in credit risk factor. It also helps them to calculate how much capital they need to set aside to protect against such risks. In finance, default occurs when a debtor has not met its legal obligation according to the debt contract, e.g. it has not made a scheduled payment, or has violated a loan covenant of the debt contract. Default may occur if the debtor is either unwilling or unable to pay debt. This can occur with all debt obligations including bonds, mortgages, loans, and promissory notes.

The risk of loss arising from a debtor being unlikely to pay its loan obligations in full or the debtor is more than 90 days past due on any material credit obligation; default risk may impact all credit-sensitive transactions, including loans, securities and derivatives. Default probability is the likelihood that the counterparty will default on its obligation either over the life of the obligation or over some specified horizon, such as a year

The general idea of my study is to analyze the actual effect on distance default and probability of default. In the Merton model, default occurs when the "surprise" term d_2 is large enough (typically a large negative number). In the numerator, $\ln\left(\frac{V}{X}\right)$ (see equation 9) is the actual continuously compounded return on the assets that is necessary to lead to default.

If $X > V$, this return is negative (i.e., the asset value must fall to lead to default). The term $\left(r - \frac{\sigma^2}{2}\right)T$ (see equation 9) is the expected value of the continuously compounded return (usually positive). Thus the numerator is the difference between the actual continuously compounded rate of return required for default and the expected value of the return, i.e., it is the "surprise", or unexpected component of the rate of return necessary for default. The denominator is the standard deviation of the rate of return.

Therefore, the ratio (again typically negative) measures the number of standard deviations of return necessary to lead to default at time T . The negative of this ratio (a positive

number) is called the distance-to default. Distance to default is smaller (and default probability higher) when volatility is higher and maturity is longer.

3. Black Schools Model

For a continuous process, price changes becomes smaller as time periods get shorter, the binominal model for pricing the options (call and put option) converges on the Black Scholes model. It is also known as Black Scholes Merton model because the model got name after its co-creators. I.e. Robert Merton, Myron Scholes and Fischer Black [10, 4, 1].

The Black Scholes model is basically a mathematical formula that is used to calculate the European call option and put option [4] but in this paper we are using only European call option.

The formula of European call option is [8]

$$C(S, T) = SN(d_1) - Ke^{-rT}N(d_2) \quad (4)$$

Where,

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (5)$$

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \quad (6)$$

S is the present price of the stock, K is the strike price, r is the free risk rate interest, σ is the variance parameter of the stock and N is the CDF function for a standard normal distribution.

In this paper we are going to use BSM for call option (European) to find the probability default of the firm. The d_2 is very important because it is the idea behind the Merton model and $N(-d_2)$ is the idea behind the probability default.

4. Merton Model

Merton model (1974) was the first model that describes the default and it is considered the first structural model. Merton's model assumes that a firm has issued both equity as well as debt such that its total value at time T is V . This varies over a time as a result of actions by the firm, which does not pay any dividend on its equity or coupons on its bond [15].

Part of the firm's value is zero coupon debt with promised repayment of amount X at time T . At time T the remainder of the value of the firm will be distributed to the shareholders and firm will be wound up. The debt holder's rank is above the share holders in the wind up of a firm. So, provided the firm has sufficient funds to pay the debt, the share holders will receive $V - X$. The firm will be default only when the total value of the asset (V) is less than the promised debt (X) at time T . I.e. $V < X$

In this situation the bondholders will receive V instead of X and the shareholders will receive nothing. Combine the above two conditions, we see that the share holders will receive a payoff of $\max(V - X, 0)$ at time T . This can be

regarded as treating the share holders of the firm as having a European call option on the asset of the firm with expire date T and an exercise/strike price equal to the value of the debt. The Merton model can be used to estimate the probability default.

A second approach was introduced by Black and Cox (1976). In this approach default occurs when the firm's value (V) goes below a certain threshold (X). In Merton approach, default can occur at any time. Following assumptions would undermine the model efficiency [11]:

1. The firm can default only at time T and not before.
2. Assets of the firm's follow lognormal distribution.
3. Default probability for private companies can be estimated only based on the accounting data.
4. The model does distinguish between the types of default according to their seniority, convertibility and collaterals.

5. Merton Model vs. Black Scholes Model for European Call Option

The Black Scholes model used first by Merton (1974) who applies the option pricing formula of Black Scholes model to find the firms default. In Merton's model, the firm's capital structure is assumed to be composed by equity and a zero-coupon bond with maturity T and face value of X .

The firm's equity is simply a European call option with maturity T and strike price X on the asset value and, therefore, the firm's debt value is just the asset value minus the equity value. This approach assumes a very simple and unrealistic capital structure and implies that default can only happen at the maturity of the zero-coupon bond.

The Merton model for credit risk has three steps:

1. Use the BSM formula for call option to find the price or value of the firm's equity.
2. Using the firm's equity value we assume that the firms asset value and asset volatility, estimate the probability default (PD).
3. We are going to assume that the firm's asset price follows lognormal distribution.

Role of BSM for European Call option in Merton for credit risk:

The Black-Scholes for a European call option

$$C(V, T) = V * N(d_1) - X * e^{-rT}N(d_2) \quad (7)$$

Where,

$$d_1 = \frac{\ln\left(\frac{V}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (8)$$

$$d_2 = \frac{\ln\left(\frac{V}{X}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \quad (9)$$

In order to find the default distance and probability default of a firm we assume that:

1. S In BSM is replaced by firm asset value, V in Merton model, where $V = D + E$
2. K In BSM is replaced by firm's debt X in Merton

model, its total face value of debt because that is the "strike" that must be paid to retire debt and own the firm's assets.

3. r Is the expected growth on the firm's asset not risk free rate.

Firm's value (V) corresponds to stock price (S), Firm's value debt (X) corresponds to exercise/strike price (K) and r is the expected growth on the firm's asset not risk free rate.

Now we will illustrate the importance of $N(-d_2)$ in the BSM approach.

1. BSM for European call option is directly applied in a first step to find the value of the equity of a firm/maybe to get debt of a firm.
2. In 2nd step, $N(-d_2)$ is used to estimate the PD of a firm. We are going to assume that the risk free rate (r) in BSM is replaced with a real firm drift (μ). It is not an option pricing at all. It is just a simple statistical calculation and distance default.
3. Again, $N(-d_2)$ is the analog to probability default (PD), except real asset drift replaces risk free rate

Example (1)

Let us consider a simple example:

Firm Value (V) = 130, FV Debt (X) = 100, Expected return (μ) = 5%, Time (T)=1 Period, Volatility (σ) = 20%

The Black Scholes formula for European call option is (equation 4, 5, and 6)

$$C(V, T) = V * N(d_1) - X * e^{-rT} N(d_2) \tag{10}$$

Where

$$d_1 = \frac{\ln(\frac{V}{X}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \tag{11}$$

$$d_2 = \frac{\ln(\frac{V}{X}) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \tag{12}$$

V Is the Firm value, X is the FV debt, r is the expected return, σ is the variance parameter of the stock and N is the CDF function for a standard normal distribution.

Here the useful information is given in a d_2 of the BSM approach. We are discussing only d_2 because d_2 in BSM relates to Morton model in credit default risk. However, one of the useful and interesting is $N(-d_2)$ is the probability that this option (call) stuck in the risk neutral world not a real world.

So,

$$d_2 = \frac{\ln(\frac{S}{K}) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \tag{13}$$

d_2 Is the inside of standard normal cumulative distribution function in the BSM approach and d_2 really captured the idea of the Morton model (credit risk). Merton Model is the structural model because it is using the firms value (V) to inform the probability of default of a firm. So we go for regular option input in European call option pricing. To using inputs that concern the firm because remember the key idea of the Morton model is that we treat the firm's equity like a call option on a firm's asset. So here we have input

assumption:

Firm's value (V) corresponds to stock price (S), Firm's value debt (X) corresponds to exercise/strike price (K) and r is the expected growth on the firm's asset not risk free rate.

Now we start at time $t = 0$ the firm value $V = 130$ and the default threshold $X = 100$ as given in above example. So, even before time $t = 0$, $\ln(\frac{V}{X}) = \ln(\frac{130}{100}) = 233\%$. Just start from today, here in exist capital structural the value of the firm has dropped by 233% before default. At time $T = 1$ the end of the period, firm will have an expected future value higher than today, due to positive drift. In this case, $V_1 = 134$ as shown in figure 1. In equation (9) we can see $(r - \frac{\sigma^2}{2})$ is the geometric average.

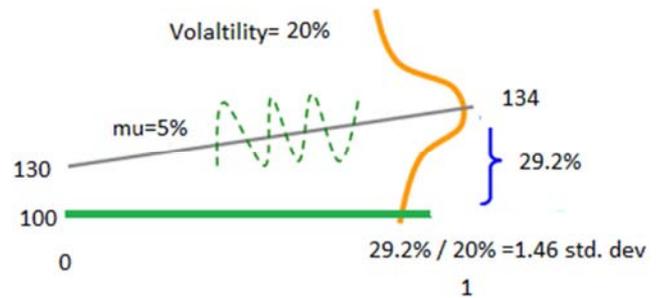


Figure 1. Probability default.

In the Merton model, default occurs when the term d_2 is large enough (typically a large negative number). In the numerator, $\ln(\frac{V}{X}) = 234\%$ (see equation 9) is the actual continuously compounded return on the assets that are necessary to lead to default.

If $X > V$, this return is negative (i.e., the asset value must fall to lead to default). The term $(r - \frac{\sigma^2}{2})T = 3\%$ (see equation 9) is the expected value of the continuously compounded return (usually positive). Thus the numerator of d_2 is the difference between the actual continuously compounded rate of return required for default and the expected value of the return, i.e., it is the "surprise", or unexpected component of the rate of return necessary for default. The denominator is the standard deviation of the rate of return. Therefore, the ratio (again typically negative) measures the number of standard deviations of return necessary to lead to default at time T .

If we are going to make a normal/lognormal assumption, we can treat either, but it is easier to treat the normal log returns. Our expected future firm value is 29.2% standard deviations above the default threshold as shown in figure 1. As our asset volatility is 20%, this implies our expected future firm value will be 1.46 sigma above the default threshold of 100.

Under this series of unrealistic assumptions, future insolvency is characterized by a future firm value that is lower than the default threshold of 100; i.e., the area in the tail.

$$PD = N(-d_2) = 7.19\%$$

Here in above example the distance to default $d_2 = 1.46$ standard deviation or 29.2% converted into standard deviation term and $N(-d_2)=7.19\%$ is the default probability of the firm. $N(-d_2) = 7.19\%$ implies that it is the probability that a firm will default. In figure 1 it is seen that the line below the threshold (yellow line below the green line is the probability default) that is probability default.

6. Factors Affecting Default Probability

The default probability affecting by various factors that is:

1. Firms value V
2. FV of debt X
3. $\mu(r)$ the expected return

4. Time period T
5. Volatility σ

Here we are using some examples so that we can easily understand the affects of “probability default” using BSM of European call option. We will change one factor and others are same in that way we will describe the factors affecting the probability default.

6.1. Firm Value (V) vs. Probability Default

Below table 1 showing the firm value increases and other factors are same we will draw a figure with the help of Excel between Firm value (V) and probability default $N(-d_2)$

Table 1. Firm Value vs. Probability Default.

Firm Value (V)	Firm Value debt (X)	“mu”(r)	Time T	volatility	d_2	$N(-d_2)$
130	100	0.05	1	0.2	1.461821	0.071895
140	100	0.05	1	0.2	1.832361	0.033449
150	100	0.05	1	0.2	2.177326	0.014728
160	100	0.05	1	0.2	2.500018	0.006209
170	100	0.05	1	0.2	2.803141	0.00253
180	100	0.05	1	0.2	3.088933	0.001004

The figure 2 shows us that if the firm value (v) increases then the probability of default will decrease because debt value is constant as shown in table 1 and the firm value is increasing year by year.

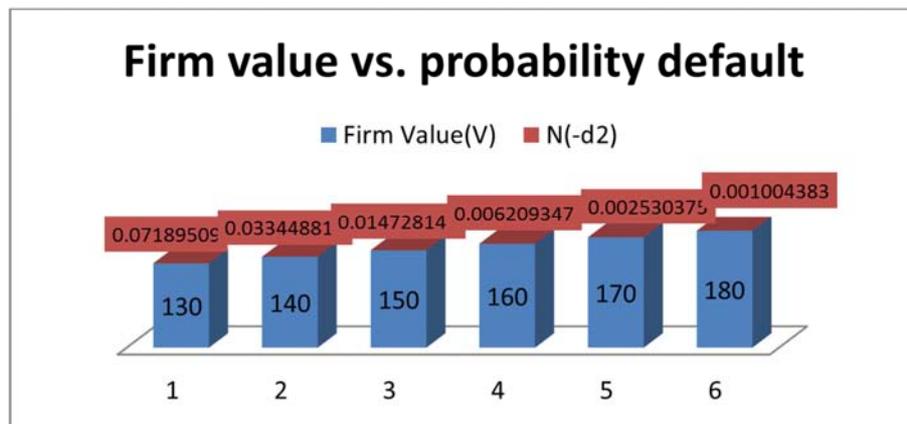


Figure 2. Firm Value vs. Probability Default.

6.2. FV of Debt (X) vs. Probability Default

Below table 2 showing the FV of debt (X) increases and other factors are same we will draw a figure with the help of Excel between FV of debt (X) and probability default $N(-d_2)$.

Table 2. FV of Debt vs. Probability Default.

Firm Value (V)	Firm Value debt (X)	“mu”(r)	Time T	volatility	d_2	$N(-d_2)$
130	100	0.05	1	0.2	1.461821	0.071895
130	105	0.05	1	0.2	1.217871	0.111637
130	110	0.05	1	0.2	0.98527	0.162246
130	115	0.05	1	0.2	0.763012	0.222728
130	120	0.05	1	0.2	0.550214	0.291086
130	125	0.05	1	0.2	0.346104	0.364632

The figure 3 shows us that if the FV of debt (X) increases (but not more than V) then the probability of default will increase because debt value is increasing every year and firm value is constant as shown in table 2.

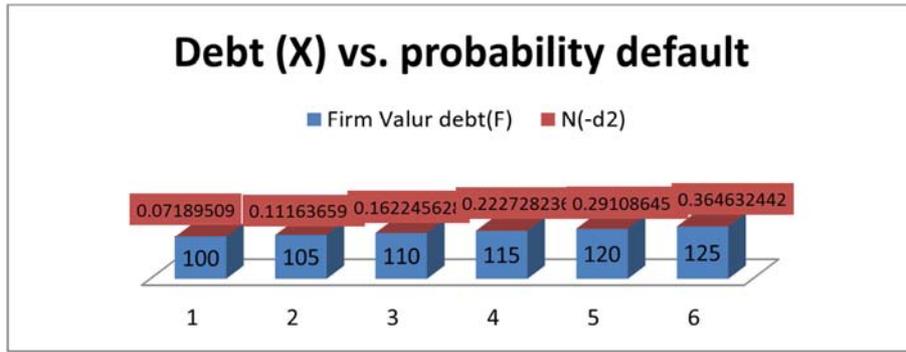


Figure 3. FV of Debt vs. Probability Default.

6.3. “Mu” (r) vs. Probability Default

Below table 3 showing the “mu” increases and other factors are same we will draw a figure with the help of Excel between “mu” and probability default $N(-d_2)$.

Table 3. Mu vs. Probability Default.

Firm Value (V)	Firm Value debt (X)	“mu”(r)	Time T	volatility	d_2	$N(-d_2)$
130	100	0.05	1	0.2	1.461821	0.071895
130	100	0.06	1	0.2	1.511821	0.06529
130	100	0.07	1	0.2	1.561821	0.059165
130	100	0.08	1	0.2	1.611821	0.0535
130	100	0.09	1	0.2	1.661821	0.048274
130	100	0.1	1	0.2	1.711821	0.043465

The figure 4 shows us that probability default decrease when “mu” increases.

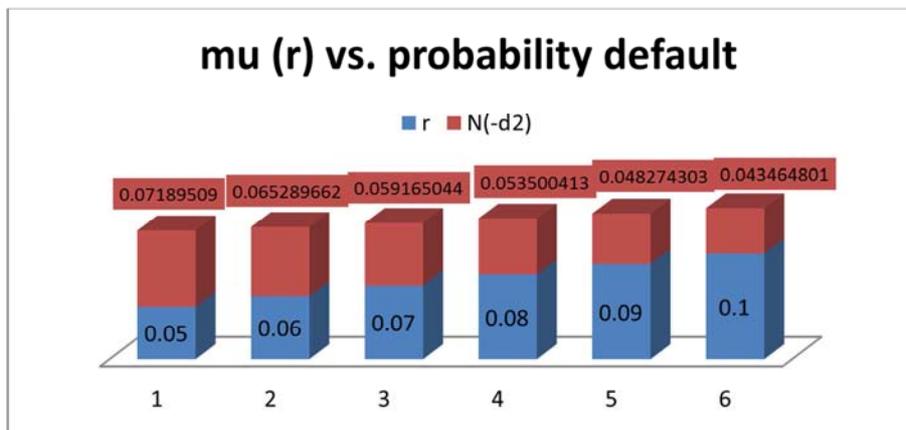


Figure 4. Mu vs. Probability Default.

6.4. Time Period vs. Probability Default

Below table 4 showing the time period increases and other factors are same we will draw a figure with the help of Excel between time period and probability default $N(-d_2)$.

Table 4. Time Period vs. Probability Default.

Firm Value (V)	Firm Value debt (X)	“mu”:r	Time (T)	volatility	d_2	$N(-d_2)$
130	100	0.05	1	0.2	1.461821	0.071895
130	100	0.05	2	0.2	1.13973	0.127199
130	100	0.05	3	0.2	1.017188	0.154532
130	100	0.05	4	0.2	0.955911	0.169559
130	100	0.05	5	0.2	0.922075	0.178245
130	100	0.05	6	0.2	0.902972	0.18327

The figure 5 shows us that probability default increase when time period increases.

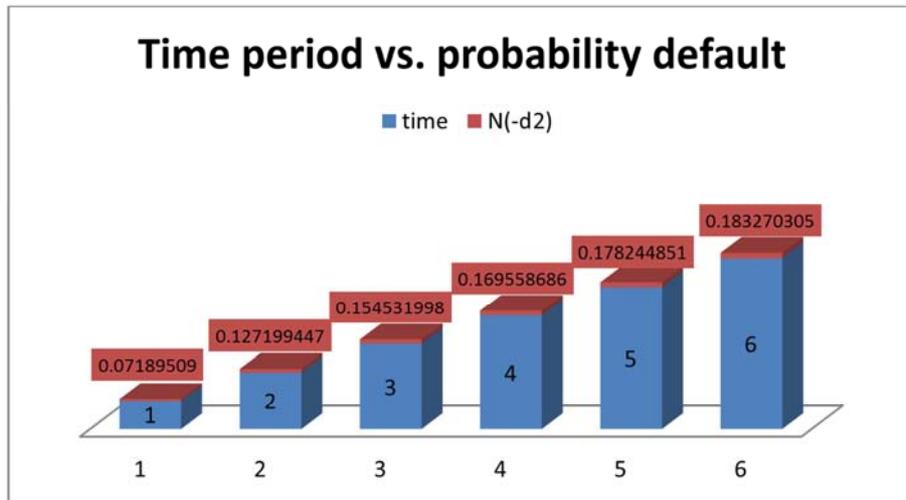


Figure 5. Time Period vs. Probability Default.

6.5. Volatility vs. Probability Default

Below table 5 showing the time volatility and other factors are same we will draw a figure with the help of Excel between time period T and probability default $N(-d_2)$.

Table 5. Volatility vs. Probability Default.

Firm Value (V)	Firm Value debt (X)	"mu" (r)	Time T	volatility	d_2	$N(-d_2)$
130	100	0.05	1	0.2	1.461821	0.071895
130	100	0.05	1	0.21	1.382449	0.083417
130	100	0.05	1	0.22	1.309838	0.095125
130	100	0.05	1	0.23	1.243105	0.106914
130	100	0.05	1	0.24	1.181518	0.118699
130	100	0.05	1	0.25	1.124457	0.18327

The figure 6 shows us that probability default increase when time volatility increases.

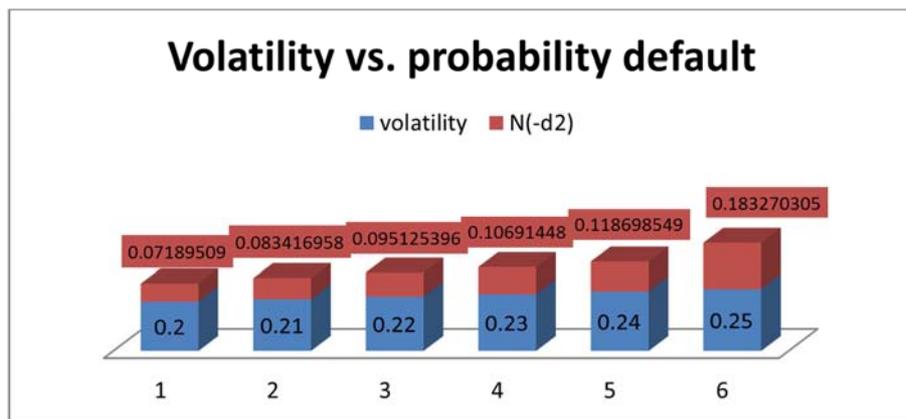


Figure 6. Volatility vs. Probability Default.

7. Conclusion

In this paper:

1. We use the BSM for European call option to find the probability default with some assumptions:
 - S In BSM is replaced by firm asset value, V in Merton

model, where $V = D + E$

- K In BSM is replaced by firm's debt X in Merton model, its total face value of debt because that is the "strike" that must be paid to retire debt and own the firm's assets.
- r Is the expected growth on the firm's asset not risk free rate.

2. We estimated d_2 that is the distance to default of a firm it gives us idea that how much a firm's value is above the threshold value.
3. $N(-d_2)$ is the default probability of the firm which the idea behind the Merton Model to Credit risk, which indicates that what percentage a firm will default.

We defined the factors that affect the probability of default using BSM for European call option, that are:

- The probability of default will decrease if the firm value (V) will increase.
- The probability of default will increase if the FV of debt (X) increases.
- If $X > V$, this return is negative (i.e., the asset value must fall to lead to default).
- The probability of default decreases when "mu" (the expected return r) increases.
- The probability of default increases when the time period increases (T).
- The probability of default increases when volatility σ increases.

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